Problem Set 1

Due: Week 3

1. If a coin is flipped and a die is rolled, what is the sample space?
   (a) S = {H,T}
   (b) S = {1,2,3,4,5,6}
   (c) S = {(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)}
   (d) S = {H,T,1,2,3,4,5,6}
2. Which of the following statements is true about events in probability?
   (a) An event is always equal to the sample space
   (b) An event is a specific outcome within the sample space
   (c) An event is a subset of the sample space
   (d) An event is independent of the sample space
3. Suppose that the events A and B are disjoint. Under what conditions are A^c and B^c disjoint?
   (a) No additional condition needed
   (b) $P(A \cup B) = S$
   (c) $P(A^c)P(B^c) = 0$
   (d) None of the above
4. Which of the following is necessary for applying the law of total probability?
   (a) The events must be independent
   (b) The events must be mutually exclusive and collectively exhaustive
   (c) The events must be conditional
   (d) The events must have the same probability
5. Two events A and B are independent if
   (a) $P(A \cup B) = P(A) + P(B)$
   (b) $P(A \cap B) = P(A) \times P(B)$
   (c) $P(A|B) = P(A)$
   (d) Both (b) and (c)
6. If A and B are independent, which of following is correct?
   (a) $P(A \cap B) = \phi$
   (b) $P(A) \neq P(B)$
   (c) A^c and B^c are also independent

(d) A^c and B^c must be disjoint

7. Drunk driving is a crime. Suppose the alcohol test has an accuracy rate of 95%. Which of the following statements is correct regarding the accuracy rate?
   (a) It means if a driver is tested positive, he is 95% likely drunk
   (b) If a driver is free of alcohol, the probability of he being tested positive is 5%
   (c) If a driver is tested positive, there is a 5% chance that he has not drunk any alcohol at all
   (d) None of the above

8. In a roulette game, the gambler has 1/10 chance of winning a prize in each spin. Jack has tried 9 times and lost them all. What of the following statements is correct?
   (a) Jack is a high probability of winning the next round
   (b) P(Winning | 9 Loses) > P(Winning)
   (c) The chance of winning stays 1/10 no matter how many times he has played
   (d) None of the above

9. A box contains 30 red balls, 30 white balls, and 30 blue balls. If 10 balls are selected at random, without replacement, what is the probability that at least one color will be missing from the selection? (Ex1.10.6)

10. Suppose that a family has exactly n children ($n \geq 2$). Assume that the probability that any child will be a girl is 1/2. Assume that all births are independent. Given that the family has at least one girl, what's the probability that the family has at least one boy? (Ex2.5.27)

Solutions

1-8: CCBBDCBC

9. Let A_1 denote the event that no red balls are selected, let A_2 denote the event that no white balls are selected, and let A_3 denote the event that no blue balls are selected. The desired probability is $P(A_1 \cup A_2 \cup A_3)$. We want to apply the formula:

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_2 \cap A_3) - P(A_1 \cap A_3) + P(A_1 \cap A_2 \cap A_3).$$

The event A_1 will occur if and only if the 10 selected balls are either white or blue. Since there are 60 white and blue balls, out of a total of 90 balls, we have

$$P(A_1) = \frac{C_{60}^{10}}{C_{90}^{10}}.$$

Similarly, $P(A_2)$ and $P(A_3)$ have the same value. The event $A_1 \cap A_2$ will occur if and only if all 10 selected balls are blue. Therefore,

$$P(A_1 \cap A_2) = \frac{C_{30}^{10}}{C_{90}^{10}}.$$

Similarly, $P(A_2 \cap A_3)$ and $P(A_1 \cap A_3)$ have the same value. Finally, the event $A_1 \cap A_2 \cap A_3$ will occur if and only if all three colors are missing, which is obviously impossible. Therefore, $P(A_1 \cap A_2 \cap A_3) = 0$. When these values are substituted into the above equation, we obtain the desired probability,

$$P(A_1 \cup A_2 \cup A_3) = 3 \times \frac{C_{60}^{10}}{C_{90}^{10}} - 3 \times \frac{C_{30}^{10}}{C_{90}^{10}} \approx 0.04.$$

10. Let A denote the event that the family has at least one boy, and B be the event that it has at least one girl. Then

$$P(B) = 1 - (1/2)^n,$$
$$P(A \cap B) = 1 - P(\text{all girls}) - P(\text{all boys}) = 1 - (1/2)^n - (1/2)^n.$$

Hence,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1 - (1/2)^{n-1}}{1 - (1/2)^n}.$$