Problem Set 2

Due: Week 6

1. Which of the following best describes a random variable?
   (a) A deterministic outcome in an experiment
   (b) A function that assigns a numerical value to each outcome in a sample space
   (c) The probability of an event occurring
   (d) A measure of central tendency in a data set
2. Which of the following is best suited to be modeled as a random variable?
   (a) The distance between the campus to the city center
   (b) The probability of winning a lottery
   (c) The lifespan of a cat
   (d) None of the above
3. Which of the following statements is true about the probability distribution of a discrete random variable?
   (a) The sum of all possible probabilities must equal 1
   (b) The possible values cannot be negative
   (c) The distribution must be symmetrical
   (d) Both (a) and (b)
4. For a continuous random variable X, the probability that X takes on a specific value is:
   (a) Given by the probability density function (PDF)
   (b) Given by the area under the PDF over an interval
   (c) Always equal to 1
   (d) Always equal to 0
5. What is the primary characteristic of a probability mass function (PMF)?
   (a) It can only be used for continuous random variables
   (b) It integrates to 1 over the entire range of the variable
   (c) It assigns probabilities to each possible value of a discrete random variable
   (d) It represents the cumulative probability up to a certain value

6. Suppose X is a random variable with the CDF:

X	1	3	5	7
CDF	0.5	0.75	0.9	1

What is $P(X = 3)$?
   (a) 0.15
   (b) 0.25
   (c) 0.5
   (d) 0.75

7. Which of the following scenarios can be modeled by a Bernoulli distribution?
   (a) The number of customers arriving at a store in an hour.
   (b) The number of heads when flipping a coin three times.
   (c) Whether a student passes or fails an exam.
   (d) The number of defective items in a batch of 10.

8. Which of the following is a key characteristic of a Binomial distribution?
   (a) It models the number of successes in a fixed number of independent Bernoulli trials.
   (b) It models the time until the first success in a sequence of independent trials.
   (c) It models the number of successes in an infinite number of trials.
   (d) It models the number of events occurring in a fixed interval of time.

9. There are 100 balls in a box, numbered 1,2,...,100 (no number appears more than once). Five balls are drawn, one at a time, *with replacement*. Answer the following questions.
   (a) What is the distribution of the number of balls with a value of at least 80?
   (b) What is the distribution of the value of the j-th draw ($1 \leq j \leq 5$)?
   (c) What is the probability that the number 100 is drawn at least once?

   Now consider random sampling *without replacement*. Answer the same set of questions.

   (d) What is the distribution of the number of balls with a value of at least 80?
   (e) What is the distribution of the value of the j-th draw ($1 \leq j \leq 5$)?
   (f) What is the probability that the number 100 is drawn at least once?

10.
   (a) Show that $p(k) = \left(\frac{1}{2}\right)^{k+1}$ for $k = 0, 1, 2, \ldots$ is a valid PMF for a discrete random variable.
   (b) Find the CDF of a random variable with the PMF from (a).