

Problem Set 3

Due: Week 9

Multiple Choice Questions (2 points each)

1. The expectation $E(X)$ of a discrete random variable X is:
 - (a) The most likely value that X can take
 - (b) The weighted average of all possible values that X can take
 - (c) The square of the standard deviation of X
 - (d) The probability that X equals its mean
2. Suppose X is a discrete random variable with the following PMF:

| | | | |
|---|-----|-----|-----|
| X | 1 | 2 | 3 |
| P | 1/4 | 1/2 | 1/4 |

Which of the following is correct?

- (a) $E(X) = 2, E(1/X) = 1/2$
 - (b) $E(X) = 2, E(1/X) = 7/12$
 - (c) $E(X) = 1/2, E(1/X) = 2$
 - (d) $E(X) = 1/2, E(1/X) = 2/5$
3. If a random variable X has a variance of 0, what can be said about X ?
 - (a) X takes multiple values
 - (b) X is a continuous variable
 - (c) X is a constant
 - (d) X has a uniform distribution
 4. Which of the following is true about $E(X)$ for a discrete random variable X ?
 - (a) $E(X) = \frac{1}{n} \sum_{i=1}^n X_i$
 - (b) $E(X) = \sum_{\text{all } x} P(X = x)$
 - (c) $E(XY) = E(X)E(Y)$
 - (d) $E(aX + b) = aE(X) + b$, where a, b are constants
 5. If X and Y are independent random variables, what of the following is true?
 - (a) $E(XY) = E(X)E(Y)$
 - (b) $E(X + Y) = E(X) + E(Y)$
 - (c) $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$

- (d) All of the above
6. Suppose that X and Y are random variables such that $Var(X) = 9$, $Var(Y) = 4$, and $Corr(X, Y) = -1/6$. Which of the following is false?
- (a) $Cov(X, Y) = -1$
 - (b) $Var(X + Y) = 11$
 - (c) $Var(X - 3Y + 4) = 51$
 - (d) None of the above
7. If X is a Binomial random variable with parameters $n = 10$ and $p = 0.5$, what is $E(X)$?
- (a) 2.5
 - (b) 5
 - (c) 7.5
 - (d) 10
8. If X and Y are independent random variables, what can be said about their covariance?
- (a) $Cov(X, Y) = Var(X)Var(Y)$
 - (b) $Cov(X, Y) = E(X)E(Y)$
 - (c) $Cov(X, Y) = 0$
 - (d) $Cov(X, Y) = 1$
9. (6 points) In the Gregorian calendar, each year has either 365 days (a normal year) or 366 days (a leap year). A year is randomly chosen, with probability $3/4$ of being a normal year and $1/4$ of being a leap year. Find the mean and variance of the number of days in the chosen year.
10. (4 points) A group of 50 people are comparing their birthdays (assume their birthdays are independent, and there are 365 days in a year). Find the expected number of *pairs* of people with the same birthday.