# Problem Set 3

Due: Week 9

Multiple Choice Questions (2 points each)

1. The expectation $E(X)$ of a discrete random variable $X$ is:
   (a) The most likely value that $X$ can take
   (b) The weighted average of all possible values that $X$ can take
   (c) The square of the standard deviation of $X$
   (d) The probability that $X$ equals its mean
2. Suppose $X$ is a discrete random variable with the following PMF:

   | X | 1 | 2 | 3 |
   |---|---|---|---|
   | P | 1/4 | 1/2 | 1/4 |

   Which of the following is correct?
   (a) $E(X) = 2, E(1/X) = 1/2$
   (b) $E(X) = 2, E(1/X) = 7/12$
   (c) $E(X) = 1/2, E(1/X) = 2$
   (d) $E(X) = 1/2, E(1/X) = 2/5$
3. If a random variable $X$ has a variance of 0, what can be said about $X$?
   (a) $X$ takes multiple values
   (b) $X$ is a continuous variable
   (c) $X$ is a constant
   (d) $X$ has a uniform distribution
4. Which of the following is true about $E(X)$ for a discrete random variable $X$?
   (a) $E(X) = \frac{1}{n} \sum_{i=1}^{n} X_i$
   (b) $E(X) = \sum_{\text{all } x} P(X = x)$
   (c) $E(XY) = E(X)E(Y)$
   (d) $E(aX + b) = aE(X) + b$, where $a, b$ are constants
5. If $X$ and $Y$ are independent random variables, what of the following is true?
   (a) $E(XY) = E(X)E(Y)$
   (b) $E(X + Y) = E(X) + E(Y)$
   (c) $Var(X + Y) = Var(X) + Var(Y)$

(d) All of the above

6. Suppose that $X$ and $Y$ are random variables such that $Var(X) = 9$, $Var(Y) = 4$, and $Corr(X, Y) = -1/6$. Which of the following is false?
   (a) $Cov(X, Y) = -1$
   (b) $Var(X + Y) = 11$
   (c) $Var(X - 3Y + 4) = 51$
   (d) None of the above

7. If $X$ is a Binomial random variable with parameters $n = 10$ and $p = 0.5$, what is $E(X)$?
   (a) 2.5
   (b) 5
   (c) 7.5
   (d) 10

8. If $X$ and $Y$ are independent random variables, what can be said about their covariance?
   (a) $\text{Cov}(X, Y) = \text{Var}(X)\text{Var}(Y)$
   (b) $\text{Cov}(X, Y) = E(X)E(Y)$
   (c) $\text{Cov}(X, Y) = 0$
   (d) $\text{Cov}(X, Y) = 1$

9. (6 points) In the Gregorian calendar, each year has either 365 days (a normal year) or 366 days (a leap year). A year is randomly chosen, with probability $3/4$ of being a normal year and $1/4$ of being a leap year. Find the mean and variance of the number of days in the chosen year.

10. (4 points) A group of 50 people are comparing their birthdays (assume their birthdays are independent, and there are 365 days in a year). Find the expected number of *pairs* of people with the same birthday.

# Solutions

## Question 1-8

BBCD DDBC

## Question 9

Let $X$ be the number of days in a chosen year. Then

$$X = \begin{cases} 365 & \text{with prob. } 3/4 \\ 366 & \text{with prob. } 1/4 \end{cases}$$

By definition of mean and variance,

$$E(X) = 365 \times 3/4 + 366 \times 1/4 = 365.25,$$
$$Var(X) = E(X^2) - (EX)^2 = 365^2 \times 3/4 + 366^2 \times 1/4 - 365.25^2 = 0.1875.$$

## Question 10

Let $X$ be the number of pair matches (pairs of people with the same birthday). There are $\binom{n}{2}$ pairs in total. Order the pairs in some certain way. Define

$$I_j = \begin{cases} 1 & j\text{-th pair matches} \\ 0 & \text{otherwise} \end{cases}$$

Then we can write

$$X = I_1 + I_2 + \cdots + I_{\binom{n}{2}}.$$

We create an indicator for each pair of people since $X$ counts the number of *pairs of people* with the same birthday. The probability of any two people having the same birthday is $1/365$. That is, $E(I_j) = 1/365$ for any $j$. Therefore,

$$E(X) = \binom{n}{2} E(I_j) = \binom{50}{2} \frac{1}{365} \approx 3.36.$$