Problem Set 4

Due: Week 11

Multiple choice questions (2 points each)

1. Which of the following defines a Geometric distribution?
   (a) It models the number of successes in a fixed number of independent trials.
   (b) It models the number of trials needed to get the first success in a sequence of independent Bernoulli trials.
   (c) It models the number of events occurring in a fixed interval of time.
   (d) It models the number of successes in a Poisson process.

2. If X is a Poisson random variable with parameter $\lambda = 3$, what is $E(X)$?
   (a) 1
   (b) 3
   (c) 6
   (d) 9

3. In a factory, 10% of the products produced are defective. Let X be the random variable representing the number of defective products in a batch of 20 products. Which of the following distributions best models this scenario?
   (a) $X \sim \text{Bernoulli}(p = 0.1)$
   (b) $X \sim \text{Binomial}(n = 20, p = 0.1)$
   (c) $X \sim \text{Geometric}(p = 0.1)$
   (d) None of the above

4. A call center receives an average of 5 calls per hour. Let X be the random variable representing the number of calls received in the next hour. Which of the following distributions best models this scenario?
   (a) $X \sim \text{Bernoulli}(p = 5)$
   (b) $X \sim \text{Binomial}(n = 7, p = 5/60)$
   (c) $X \sim \text{Poisson}(\lambda = 5)$
   (d) None of the above

5. Suppose that a certain type of magnetic tape contains on average three defects per 1000 feet. What is the probability that a roll of tape 1200 feet long contains no defects?

   (a) $1 - \frac{3}{1000}$
   (b) $\left(1 - \frac{3}{1000}\right)^{\frac{1200}{1000}}$
   (c) $e^{-3.6}$
   (d) None of the above

6. Which of the following statements about joint probability distributions is true?

   (a) They describe the probability of two or more independent happened at the same time
   (b) They are composed from the probability distribution of each random variable
   (c) They describe the probability of two or more random variables occurring simultaneously
   (d) None of the above

7. Given the joint distribution $f(x, y)$ and marginal distributions $f_X(x)$ and $f_Y(y)$. Which of the following is true?

   (a) $f_X(x)$ is a probability function only about random variable X
   (b) $f_X(x) = P(X = x|Y = y)$
   (c) $f(x, y) = f_X(x)f_Y(y)$
   (d) $f(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_X(x)f_Y(y)dxdy$

8. If $f(x, y)$ represents the joint probability distribution of two discrete random variables X and Y, which of the following expressions correctly computes the marginal probability function $f_X(x)$?

   (a) $f_X(x) = f(x, y_0)$ for some specific y_0
   (b) $f_X(x) = \sum_y f(x, y)$
   (c) $f_X(x) = f(x|y)f(y)$
   (d) $f_X(x) = \int_{-\infty}^{\infty} f(x, y)dy$

9. Suppose that X and Y have a discrete joint distribution for which the joint PMF is defined as follows:

$$p(x, y) = \begin{cases} c|x + y| & \text{for } x = -2, -1, 0, 1, 2 \text{ and} \\ & \quad y = -2, -1, 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a) (3 points) Determine the value of c.
   (b) (3 points) Find the marginal distribution of X.
   (c) (4 points) Find $P(|X - Y| \leq 1)$

Solutions

1. B
2. B
3. B
4. C
5. C
6. C
7. A. B confuses marginal distribution with conditional distribution. C is wrong because we do not assume independence. D is nonsense.
8. B. C is correct but it is not how we compute marginal distribution. D is correct for continuous distributions, but here we have discrete distributions.
9. Solution:

   (a) Let's list all the combinations of x and y:

Y\X	-2	-1	0	1	2
-2	4c	3c	2c	c	0
-1	3c	2c	c	0	c
0	2c	c	0	c	2c
1	c	0	c	2c	3c
2	0	c	2c	3c	4c

   All the probabilities in table must sum up to 1, that is $40c = 1$. Therefore, $c = 1/40$.

   (b) To find the marginal distribution of X, sum up the rows in the table for each value of x:

X	-2	-1	0	1	2
-2	10c	7c	6c	7c	10c

   Therefore,

$$p_X(x) = \begin{cases} 1/4 & x = \pm 2 \\ 7/40 & x = \pm 1 \\ 3/20 & x = 0 \end{cases}$$

   (c) We sum up all value that satisfies $|X - Y| \leq 1$, which corresponds to the diagonal± 1 cells (in red). $P(|X - Y| \leq 1) = 28c = 7/10$.